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# Display modulation by Fourier transform: A preferred method

Thomas G. Fiske  
Louis D. Silverstein

**Abstract** — A preferred method for determining the grating modulation of a rear-projection display using a grating image and Fourier analysis is prescribed. This method is insensitive to spatial image noise and is in better correspondence with the response of the human visual system than is the standard technique. This method is not limited to rear-projection displays and can be applied to any display technology.

**Keywords** — *Display modulation, Fourier transform, image quality, projection display.*

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## 1 Introduction

The Modulation Transfer Function (MTF) or grating modulation versus spatial frequency of a display system is commonly used as a metric of the ability of the system to faithfully reproduce information as a function of spatial frequency. The data are often gathered by using a scanning slit device to record the spatial variation of luminance across a grating image. A properly calibrated CCD-based imaging device can also be used.

The maximum value and minimum value of the resulting luminance pattern are estimated, and the Michelson contrast or modulation<sup>1</sup> is calculated by using Eq. (1):

$$\frac{L_{\max} - L_{\min}}{L_{\max} + L_{\min}} \quad (1)$$

This direct method for estimating grating modulation works well for displays that present a relatively well behaved and regular spatial pattern. If the display presents an image that is relatively noisy – even on a subpixel level – accurate results are difficult to achieve with this method. In addition, while sine-wave gratings have typically been used for CRTs and analog imaging systems, square-wave gratings are often specified for direct-view matrix displays<sup>1</sup> and matrix image sources for projection systems. If the intended square-wave spatial pattern departs from a 50% duty cycle or if it has a shape that is significantly different than a square-wave (an outcome that is highly likely for projection displays based on matrix image sources), this method will give results that either over or underestimates the perceptually relevant modulation.

We propose a Fourier analysis method as a preferred alternative to the standard method. The same grating images are used. In this method, the 2-D modulation spectrum is computed from a 2-D image. The calculation is based on 2-D FFT analysis implemented in Matlab.<sup>2</sup> One could also use a 1-D analysis performed on a high-resolution slit scan-type measurement.<sup>3</sup> The amplitude of the fundamental sinusoidal component of the Fourier transform (suitably normalized) is reported as the modulation.

The Fourier analysis method has many advantages. A display with a noisy image makes the  $L_{\min}$  and  $L_{\max}$  values of the pattern – and therefore the modulation – difficult to determine. The Fourier analysis method is immune from such a difficulty as long as the noise is uncorrelated with and is in a different spatial frequency regime than the grating test pattern.

It is well established that the human visual system (HVS) analyzes spatial patterns by decomposing the patterns into their sinusoidal frequency components,<sup>4</sup> *i.e.*, its response is correlated with the spatial frequency content of a presented image as determined by the HVS contrast sensitivity function. For displays where a square-wave grating test image significantly departs from a square-wave shape or a 50% duty cycle (as might occur in a microdisplay-based projection system), the Fourier content of the grating image will obviously differ from that of a nominal square-wave grating. The HVS will respond to the differences, and an observer will report a corresponding difference in perception. The Fourier method will accurately capture the changes in spatial frequency content and reflect any perceptual differences if they exceed the frequency-dependent thresholds of the HVS. The standard method, on the other hand, will yield a modulation result that is insensitive to any perceptually significant changes in shape or duty cycle as long as  $L_{\min}$  and  $L_{\max}$  do not change.

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## 2 Modulation by direct method

Figure 1 shows views of a one-line-on/one-line-off grating image and a five-line-on/five-line-off grating image. Figure 2 shows the resulting luminance *vs.* distance plots; the data are averaged across the non-modulated direction. The data are from a prototype three-chip LCOS-based rear-projection display for avionics applications with addressable resolution of about 128 dpi.<sup>5</sup> The screen is a 5-mil bulk diffuser. The data are obtained by a ProMetric Color 1421-1 measurement system from Radiant Imaging.<sup>6</sup> The field of view is 0.573 in. in the horizontal and 0.382 in. in the vertical, and

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T. G. Fiske is with Rockwell Collins – San Jose, 2701 Orchard Parkway, MS 69, San Jose, CA 95134-1042, U.S.A.; telephone 408/532-4986, fax 408/954-1042, e-mail: [tgfiske@rockwellcollins.com](mailto:tgfiske@rockwellcollins.com).

L. D. Silverstein is with VCD Sciences, Inc., Scottsdale, AZ, U.S.A.

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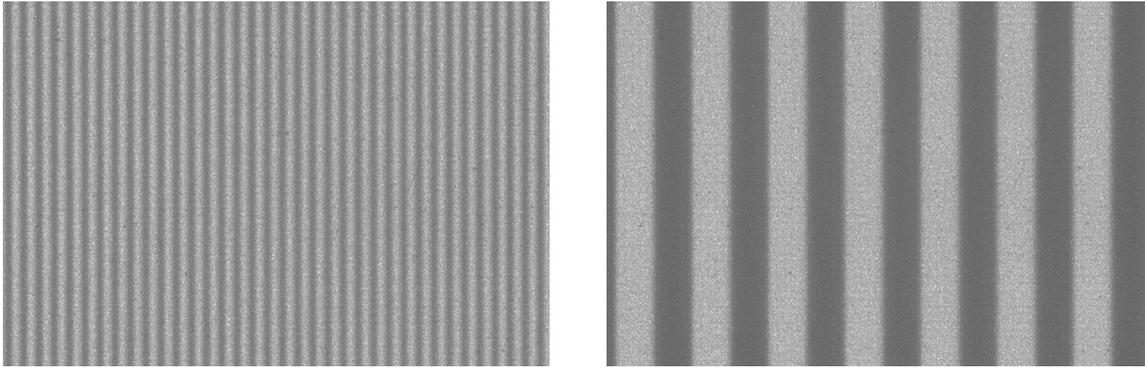


FIGURE 1 — One-line (left) and five-line (right) grating images for a rear-projection display.

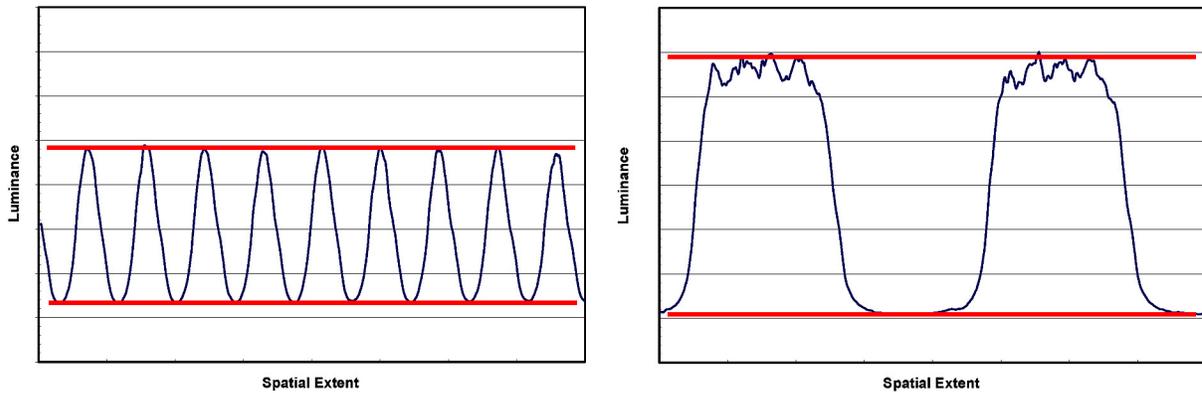


FIGURE 2 — Luminance vs. distance for the one-line (left) and five-line (right) grating images. The data is averaged across the non-modulated direction. The heavy red horizontal lines indicate the maximum and minimum luminance values.

for this particular display there are 22 sensor pixels per display pixel. The maximum and minimum luminance values are shown in Fig. 2 and the resulting modulation values are 0.55 for the one-line grating and 0.73 for the five-line grating as calculated with Eq. (1). The measurement set-up is shown in Fig. 3.

As always, the effects of veiling glare on the measurement system should be accounted for in a measurement such as this.<sup>1</sup> In this particular configuration of detector, lens, and measurement distance, the veiling glare reduces the modulation transfer factor for a high-spatial-frequency grating by about 10%. This was determined by measuring the spatial luminance distribution of a uniformly backlit ronchi ruling (*e.g.*, 50 lpi, part no. G56-592 from Edmund Optics<sup>7</sup>) that is similar in spatial frequency to the display grating of interest. We can make stray-light corrections on the full grating image by making some simplifying assumptions. An image of the full grating was acquired and compared to an image where all but one line was masked. We use the bright and dark parts of the one-line image to establish luminance correction factors for the full image. If we assume that the correction factors are uniform across the image and depend only on the luminance value at any particular point, we can apply them in a fairly straightforward manner. However, one must take care to come up with correction factors for every spatial-frequency regime and every lens and f-stop used for data acquisition. Since the effects of

veiling glare do not bear on the main issues addressed in this paper, it will not be discussed further.

### 3 Modulation by Fourier analysis

When using the Fourier analysis technique, one must take care to only include an integral number of cycles in the image

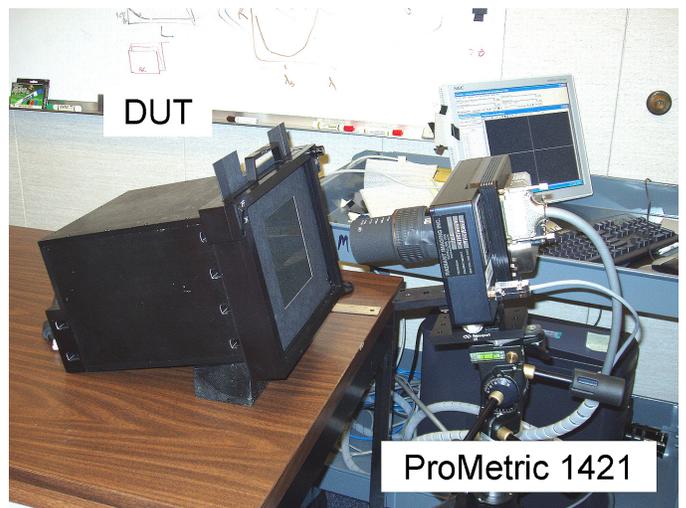
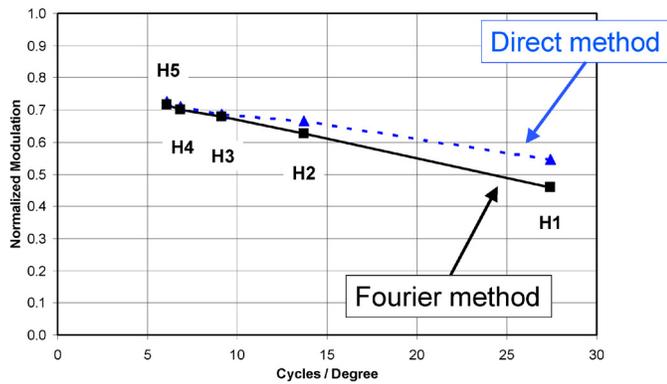


FIGURE 3 — Measurement configuration showing the ProMetric 1421 and a rear-projection display.



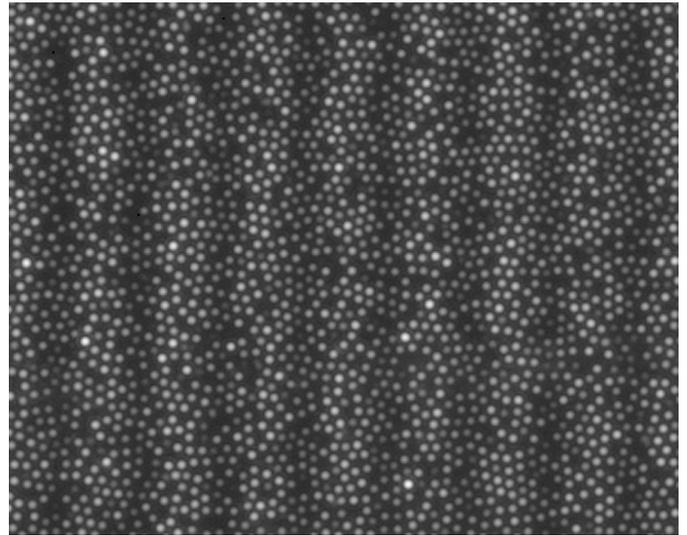
**FIGURE 4** — Modulation for the grating images of Figs. 1 and 2. The blue dashed line is direct-method modulation; the black solid line is Fourier method modulation. Hx refers to the number of lines in the grating image.

data to be analyzed. If this is not done, the extra partial cycle will lead to spurious results in calculating the amplitude of the fundamental sinusoidal component of the grating image. If one wants to calculate a modulation transfer factor as opposed to modulation of the displayed grating, the resulting amplitude must be normalized by a factor of  $4/\pi$ , which is the amplitude of the fundamental sinusoidal component of the square-wave input. When the Fourier technique is applied to the data in Fig. 2, the normalized modulation is 0.46 and 0.72 for the one-line and five-line grating, respectively.

Figure 4 shows the normalized modulation data for gratings of up to five lines calculated with both techniques. The horizontal axis is in cycles per degree calculated for a viewer at 25 in. Notice that the normalized modulation values coincide quite well at low spatial frequencies (three-, four, and five-line gratings). The values begin to depart at higher spatial frequencies (one- and two-line gratings) with the direct method overstating the normalized modulation by about 20% compared to the Fourier analysis method for a one-line grating. This difference is significant in that the Fourier result is preferred as being more perceptually meaningful.

The difference in modulation values between the methods at higher spatial frequencies is due to a couple of factors: the detailed waveform shape and the duty cycle of the waveform. In general, for waveforms with the same maximum and minimum values, the one with the more sinusoidal shape will have a smaller normalized modulation as computed by the Fourier method compared to that computed by the direct method. The modulation calculated *via* the direct method does not change when the shape of the waveform changes. The modulation calculated with the Fourier method does track the shape changes of the waveform.

To illustrate this, consider the situation where a square-wave input is applied to a display system with two different configurations, A and B. Configuration A yields a square-wave output and configuration B yields a sine-wave output. Assume also that the two curves each have the same



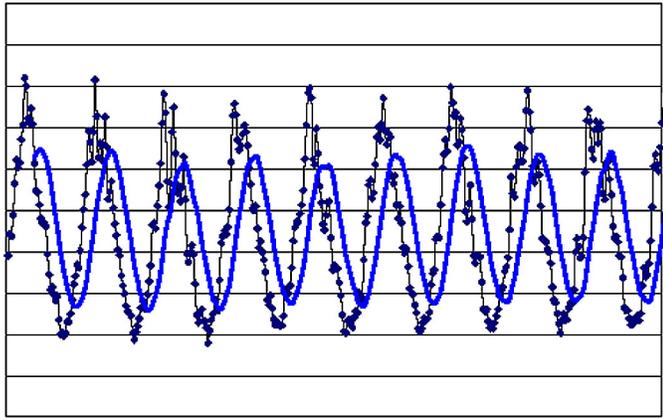
**FIGURE 5** — Magnified view of a one-line grating image on a beaded projection screen.

frequency, an amplitude of 1.0 and an offset of 0.0, and therefore have the same minimum and maximum values. The modulation of the square wave in A as calculated by the direct method is 1.0. The normalized modulation calculated by the Fourier method is also 1.0: the fundamental sinusoidal component of the Fourier transform ( $4/\pi$  for a square wave) divided by the normalization factor of  $4/\pi$ . The modulation of the sine wave in configuration B as calculated by the direct method is, again, 1.0. The normalized modulation calculated by the Fourier method is  $1.0/(4/\pi) = 0.79$ . The point here is that the direct method of calculating the modulation is insensitive to the shape difference of the waveforms, while the Fourier method does yield a result that corresponds to the shape of the waveform. The experiments by Campbell and Robson<sup>4</sup> clearly demonstrate that the HVS does respond to waveforms of different shape in proportion to the amplitudes and visual sensitivities for their sinusoidal components. The Fourier method of calculating the modulation captures this difference, the direct method does not.

Figure 5 shows a one-line grating image for a beaded projection screen. The optically relevant portion of the screen is a layer of small glass beads embedded in a black material. The structure of the screen is such that it provides for excellent ambient-light rejection. The bead size and spacing are typically many times smaller than a display pixel

**TABLE 1** — Direct modulation vs. smoothing level for the data in Fig. 6, and the modulation from the Fourier method.

Smoothing level	Min	Max	Modulation
None	78	21	0.58
¼ pixel	70	22	0.52
½ pixel	65.5	23	0.48
¾ pixel	64.5	25.5	0.43
1 pixel	59	30	0.33
Fourier			0.40



**FIGURE 6** — Luminance vs. distance for the one-line grating image on a beaded projection screen. The smooth blue line is the 3/4-pixel-wide moving average.

so the beads provide a random high-frequency sampling of the image. The luminance data are averaged across the non-modulated direction to give luminance vs. position data in Fig. 6. We see that the beaded structure of the screen results in a waveform that is quite noisy. The noise from the spatial distribution of the beads is beyond visible spatial frequencies at a typical viewing distance of 25 in., but it does cause a problem in attempting to assign maximum and minimum luminance values in order to calculate a modulation *via* the direct method. One can attempt to perform a moving average over small distances in order to reduce the noise without affecting the modulation. The problem comes in trying to determine how much averaging is enough. Table 1 shows the results of the direct modulation calculation as a function of moving average window width as well as the result of the Fourier method. We see that the direct modulation method gives a wide range of results depending on how much averaging is done. The Fourier method gives a result that is near the low end of the range of results from the direct method. Since the frequency of the spatial noise is much greater than the frequency of the grating, the contribution to the modulation due to each is easily distinguished in the Fourier method. In the case where some element of the optical system imparts enough noise to the image to make it difficult to determine the modulation by the direct method – whether or not the noise is visually perceptible and degrades the modulation – the Fourier method gives unambiguous results. This feature, as well as the uncertainty in perceptually significant modulation due to the shape of the waveform for this high-frequency grating pattern makes the use of the direct method problematic. This case highlights the advantages of the Fourier method for obtaining reliable and unambiguous results for display modulation.

## 4 Conclusion

The Fourier method for calculating display modulation provides a robust, non-ambiguous, and perceptually meaningful characterization of the spatial image quality of a display.

The Fourier method is preferred over the traditional direct method of calculating grating modulation since it correctly handles spatial noise in the waveform and can be directly related to the spatial frequency sensitivity of the human visual system.

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**Thomas G. Fiske** received his Ph.D. in condensed matter physics from the University of California at Davis. He has subsequently held staff scientist or engineering positions at Xerox PARC, dpiX, Philips Electronics, and Optiva before joining Rockwell Collins in 2002. He has been involved in display metrology, optics, and LCD technology for most of his career. He has authored over 20 papers and has been awarded over 15 patents in the area of display technology. Since 1995, he has served on the Display Measurements Subcommittee of the Program Committee for the Society for Information Display International Symposium – chairing the subcommittee from 2001 to 2003. His current focus at Rockwell Collins is on projection technology, optics, and display-measurement issues.



**Louis D. Silverstein** is the founder and Chief Scientist of VCD Sciences, Inc., an organization involved in R&D in applied vision, color science, and display technology. Since founding VCD Sciences, Inc. in 1990, he has been involved in applied vision and display research projects at over 20 corporations and U.S. government research laboratories. Prior to founding VCD Sciences, Inc., he was a Senior Research Fellow at Honeywell’s Systems and Research Center and a Research Scientist at the Boeing Company. He is a member of numerous technical societies and is a Fellow of the SID. He currently serves on the U.S. National Committee of the C.I.E. and has served an appointment to the U.S. National Academy of Sciences Committee on Vision. He is a past program and general chair of the SID international symposium, has served as an Associate Editor of the *Journal of the Society for Information Display* and on the editorial board of the journal *Human Factors*, and currently serves on the editorial boards of the journals *Color Research and Application* and *Displays*. Dr. Silverstein is the author of over 120 journal articles, book chapters, technical papers, and technical reports and has been awarded 30 patents on advanced display technology. Dr. Silverstein was the 2004 recipient of the MacBeth Award from the Inter-Society Color Council (ISCC) for outstanding contributions to the science of color and in 2004 also received a Special Recognition Award from the Society for Information Display for his pioneering contributions to the transition of

aircraft instrumentation from electromechanical to electronic displays. He was awarded a Honeywell Corporate Technical Achievement Award in 1989 for the development of optical and visual simulation methods for color matrix displays and LCDs, and in 1983 he received the Alexander C. Williams Jr. Award from the Human Factors Society for outstanding contributions to the design of a major operational system